NOTISER

Remarks on the paper by Hans Ramberg: Relation between external pressure and vapor tension of compounds, and some geological implications.[1]

Ramberg (l. c. p. 6) introduces P in his eq. (1)

$$D = D_0 \cdot e^{\frac{MX}{RT}}$$

by means of $P = X\sigma$ (we here disregard D). This cannot possibly lead to his eq. (3), where P, at fixed values of X, is supposed to vary independently. If we write eq. (1) as follows

$$D = e^{\frac{MX}{RT} + \ln D_0}$$

we see that the only possible way of introducing the external pressure as an independent variable is to make D_0 a function of it. Since the part of the pressure represented by $P = X\sigma$ is fixed with X, we make D_0 a function of the excess pressure p over this value:

$$D = e^{\frac{MX}{RT} + f(p)}$$

The special form of the function f required by eq. (3) does not follow from these considerations, and therefore it is not possible to derive eq. (3) from eq. (1) in a logical way. The reason is that eq. (1), although giving the variation of the vapor tension in a homogeneous solid exposed to its own weight, cannot possibly account for the effect of any existing pressure superimposed on the system; such a pressure may be expected to modify D_0, the vapor pressure at the top of the solid (see also below), but this is not required by eq. (1). When Ramberg writes down his eq. (3) it is therefore really a statement independent on eq. (1). In section (B) (l. c. pp. 8 et seqq.) it is eq. (3) that is verified by its compatibility with the Clausius—Clapeyron relation; eq. (1), with P introduced as done by Ramberg, cannot be verified in this way. But eq. (3) has been derived independently by H. Wergeland, according to the footnote l. c. p. 7, and in addition it is

[1] Vid.-Akad. Avh., I, 1944, No. 3.

equivalent to the equation by Nernst referred to in the footnote l. c. p. 5[1], so it is well established.

Ramberg proves (l. c. p. 6) that the vapor tension in a permeable solid must satisfy eq. (1), as otherwise a circular process would take place. Therefore it might be thought that a distribution of the vapor tension according to eq. (3) would in general give rise to a circular process and thus be impossible. However, it is easily shown that this is not the case. In eq. (3) we split up P into the part due to the weight of the solid itself, and a part applied from without ($P = X\sigma + p$):

$$D = D_0 \cdot e^{\frac{MP}{RT\sigma}} = D_0 \cdot e^{\frac{M(X\sigma + p)}{RT\sigma}} = D_0 \cdot e^{\frac{Mp}{RT\sigma}} \cdot e^{\frac{MX}{RT}}$$

This shows that eq. (3) takes the form of eq. (1) if the appropriate value of D_0 is used, namely the original D_0 multiplied by a function of p. This function is greater or less than 1 according as p is positive or negative. Eq. (1) is the special case of eq. (3) for $p = 0$. Therefore eq. (3) (including the special case eq. (1)) satisfies the equilibrium condition which prevents a circular process.

It follows that Ramberg's highly interesting discussion in section (D) (l. c. pp. 15 et seqq.) obtains a more general validity, as it is not necessary that any of the layers A or B should reach the surface. The whole system may be at any depth and have any load (not approaching critical values) placed on top of it.

I do not intend to enter upon the question of actual application of Ramberg's results in geology, as I am afraid it would require a rather lengthy discussion. But I may state as my personal view that the conditions required to start and maintain a metamorphic differentiation process on Ramberg's principles must be very rarely if at all realised in the Earth's crust, especially in the deeper zones, where metamorphic differentiation is supposed to take place. The difficulty is that the vapor pressure within the solid must literally be in accordance with eq. (3), as if an open channel were left for the vapor throughout the vertical length of the system considered. It is certainly hard to imagine such a kind of permeability in a solid (or plastic) rock.

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[1] In the present case we may write Nernst's equation

$$\frac{RT}{D} \cdot \frac{dD}{dP} = \frac{M}{\sigma}$$

which, rearranged and integrated, gives

$$D = k.e^{\frac{MP}{RT\sigma}}$$

The integration constant k is the value of D when $P = 0$, that is D_0, and we have

$$D = D_0 \cdot e^{\frac{MP}{RT\sigma}}$$

which is Ramberg's eq. (3).

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