INVESTIGATION OF ELEMENT-SPECIFIC REFLECTED X-RAYS OF THE OLDER LANGØ-GUMØ GABBRO, KRAGERØ ARCHIPELAGO, SOUTH NORWAY

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A simplified method of collecting geochemical data has been used together with trend-surface analysis and correlation analysis to study the relation between the two parts of the dissected Langø-Gumø gabbro. The results show that for Si and Fe a 2nd degree polynomial surface gives a satisfactory description of the relation between concentrations and geographical coordinate, and for Ti a 4th degree polynomial surface gives a good description. The correlation survey confirms the statement that there is no significant geochemical difference between the two parts of the gabbro as far as the elements Si, Ti, Al, Fe, Ca, and K are concerned.

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Introduction

By increasing use of instrumental chemical analysis in geochemistry, sample preparation and data handling have become the bottleneck rather than the analysis itself. This paper gives a simplified method of study whereby the physical property measured, rather than the calculated concentration, is used for the trend-surface analysis and the correlation survey.

The main foliation of the meta-sediments in the studied area is given together with major dislocation zones in Fig. 1. The Langø-Gumø gabbro is situated between two dislocation zones, one running along the coast, the other one running from Kragerø to Arø. The northern part has moved eastwards, the southern part westwards. This movement is accompanied by a compression of the whole area leading to the formation of several sharp folds between the dislocation zones. The Langø-Gumø gabbro is situated between two fold flanks, and cut by a significant dislocation zone.

1.5–2.5 kg specimens were collected according to the quadratic grid point system given in Fig. 2. The data used in the present study are drift adjusted element specific X-ray intensities collected in a Siemens X-ray spectrometer from 74 fused samples. The X-ray data, not given in this paper, are available upon request. The operating conditions of the spectrometer are given in Table 1. Each sample was counted ten times and a standard was counted for

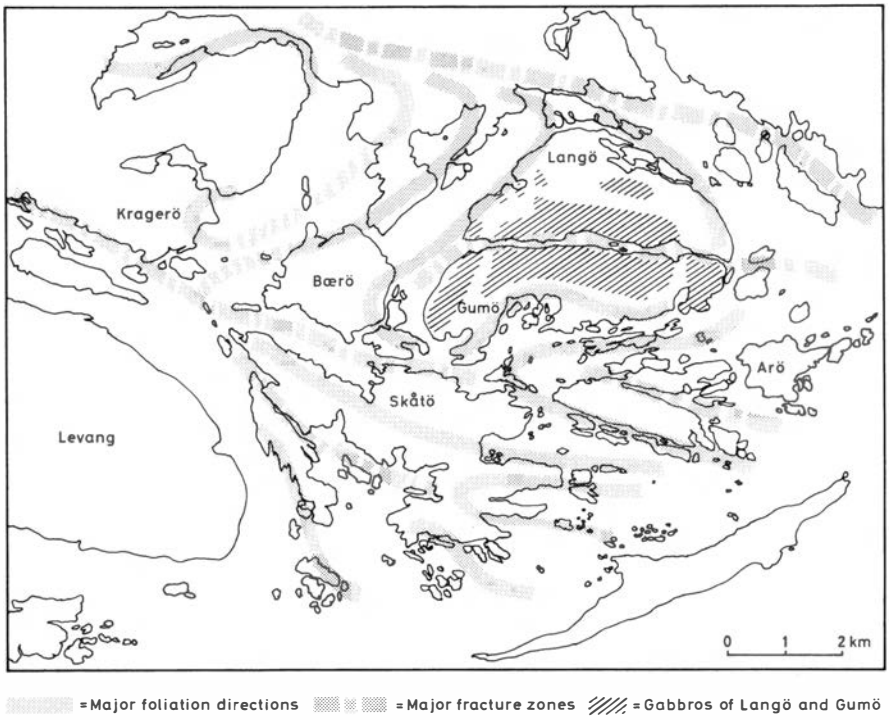

Fig. 1. Tectonic sketch of the Kragerø Archipelago.

every ten samples to keep check of the instrumental drift. The intensity data of the samples were corrected according to the drift of this standard count.

The intensity data cannot be directly related to chemical composition unless changes in the matrix effect may be disregarded or are small. For fused samples the matrix effect is mainly due to changes in chemical composition since mineral effects are non-existent. The variation in chemical composition of the present collection of samples is moderate and, following e.g. Christie & Bergstøl (1968), the variation in the matrix effect would be small. It is

Table 1. Instrumental parameters of X-ray fluorescence spectrometer.

Element	Tube	kV	mA	Crystal	Gas	Diagn. line	count. time
Si	Cr	50	40	PET	Propane	2Θ 109.18	24 sec
Ti	Cr	50	40	LiF	A/CH_4	86.12	24 sec
Al	Cr	50	40	PET	Propane	145.15	60 sec
Fe	Cr	50	40	LiF	Air	57.46	24 sec
Ca	Cr	50	40	PET	A/CH_4	45.15	24 sec
K	Cr	50	40	PET	A/CH_4	50.56	24 sec

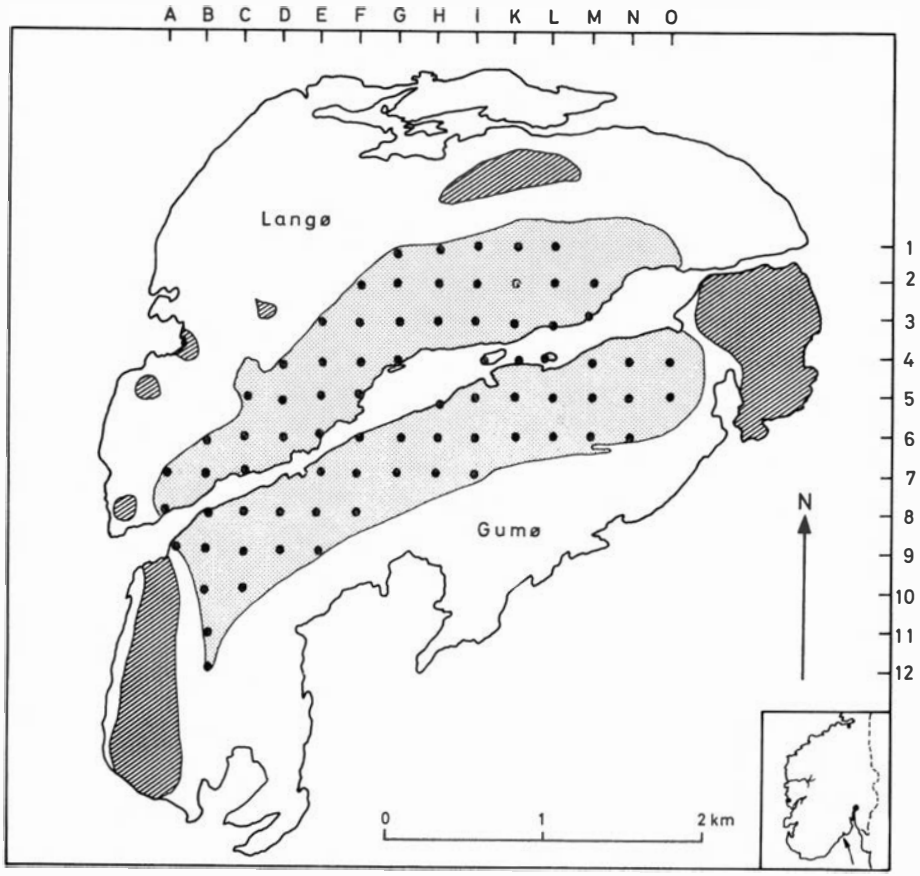

Fig. 2. Sample stations marked by dots, younger gabbros not studied are marked by hatched areas. Gabbro locations of Langø after Wiik (1962).

therefore believed that the picture obtained from the X-ray intensity data is applicable to chemical composition as well.

For chemical analyses and detailed petrographic description the reader should consult the papers by Brøgger (1934) and Wiik (1962).

Statistical analysis

The statistical analysis is based on 40 samples from Gumø and 34 samples from Langø. For each sample the following are observed:

(i) the geographic (x,y)-coordinate;
(ii) the X-ray intensity for the elements Si, Ti, Al, Fe, Ca, K.

In the first part of the analysis each element is considered separately.

Consider an arbitrary element. Let Z_{ik} = X-ray intensity of the element from sample no. k from island no. i, where i=1 for Gumø, i=2 for Langø, and k=1, . . .,n_i, where n_1=40 and n_2=34.

We shall make the following assumptions:

(1) that all Z_{ik} are independent random variables which are normally distributed;
(2) that all Z_{ik} have the same variance σ^2;
(3) that Z_{ik} is dependent on location, i.e. on geographic coordinate.

The expectation of Z_{ik}, which we denote by ζ_{ik}, depends on the geographic coordinates (x_{ik}, y_{ik}) of the sample. We shall assume that this dependence can be described by a polynomial in x_{ik} and y_{ik}.

Let its degree be g and let $t_{iko}=1$, $t_{ik1}=x_{ik}$, $t_{ik2}=y_{ik}$, $t_{ik3}=x_{ik}y_{ik}$, . . ., $t_{ikr}=y_{ik}^g$ (when for instance g=2, then r=5). Then we suppose

$$(3)\quad \zeta_{ik} = \beta_{io} + \beta_{i1}t_{ik1} + \ldots + \beta_{ir}t_{ikr},$$

where $\beta_{i0}, \beta_{i1}, \ldots, \beta_{ir}$ are the regression coefficients. The assumptions imply that the gabbro may be described by two different polynomials of the same degree.

Equality of the Langø and the Gumø polynomials

Our first aim is to test whether the two polynomials are equal, that is, we wish to test the hypothesis

$$H_o : \beta_{10} = \beta_{20}, \quad \beta_{11} = \beta_{21}, \ldots, \quad \beta_{1r} = \beta_{2r}$$

Our model, given by (1), (2), and (3), is a special case of the general linear-normal model. The method for testing H_o is well known (e.g. Sverdrup 1967, p. 206).

Let

$$Q = \sum_{i=1}^{2} \sum_{k=1}^{n_i} (Z_{ik} - \zeta_{ik})^2$$

and let Q_a be the minimum of Q under the restrictions (3) and Q_H the minimum of Q under the restrictions (3) and H_o.

We test the equality by the following method:

$$(4)\quad \text{reject } H_o \text{ if } \frac{Q_H - Q_a}{Q_a}\,\frac{n-2r-2}{r+1} > f_\varepsilon;\ r+1,\ n-2r-2$$

where the number on the right side is the upper ε-point in the Fisher distribution with r+1 and n–2r–2 degrees of freedom. The test has the significance level ε.

The formulas for Q_a and Q_H are quite simple. Let $(\hat{\beta}_{10}, \ldots, \hat{\beta}_{1r})$ and $(\hat{\beta}_{20}, \ldots, \hat{\beta}_{2r})$ be the least square estimators of the regression coefficients when Gumø and Langø are treated separately. Then

$$Q_a = \sum_{i=k}^{2} \sum_{k=1}^{n_i} (Z_{ik} - \sum_{l=0}^{r} \hat{\beta}_{il} t_{ikl})^2$$

Further, let $\hat{\hat{\beta}}_0, \ldots, \hat{\hat{\beta}}_r$ be the least square estimators of the regression coefficients when H_0 is true, based on the data from both Gumø and Langø. Then

$$Q_H = \sum_{i=1}^{2} \sum_{k=1}^{n_i} (Z_{ik} - \sum_{l=0}^{r} \beta_l t_{ikl})^2$$

The results of the computations which were carried out on an electronic computer with the aid of a standard program (NRSR) are given in Table 2.

The Gumø equations represent a family of ellipses with center in (7.42, 4.58) in the (x′, y′) system and with axes increasing with z. The Langø equations represent a family of hyperbolas with center in (6.16, –1.91) in the (x″, y″) system and asymptotes $y'' + 1.91 = 1.97\,(x'' - 6.16)$. The axes increase with z. The equations for Z values 820, 840, 860, 880, and 900 are shown in Fig. 3.

Correspondingly, the 2nd degree isopleth equation for Fe may be written

$$\frac{(x' - 10.05)^2}{0.0053(z-2644)} - \frac{(y' - 3.95)^2}{0.00543(z-2644)} = 1 \text{ (Gumø)}$$

$$\frac{(x'' - 8.87)^2}{0.0150(3175-z)} - \frac{(y'' + 0.72)^2}{0.1462(3175-z)} = 1 \text{ (Langø)}$$

where the (x, y) system is rotated 33.60° and the (x″, y″) system is rotated

Table 2. F-values and multiple correlations based on models of 2nd and 4th degrees.

Element	2nd degree				4th degree			
	F	Multiple correlations			F	Multiple correlations		
		Gumø	Langø	Both		Gumø	Langø	Both
Si	0.84	0.81	0.63	0.72	0.46	0.85	0.75	0.78
Ti	1.93	0.37	0.57	0.30	0.73	0.56	0.74	0.53
Al	1.92	0.49	0.70	0.46	0.76	0.63	0.83	0.63
Fe	0.92	0.51	0.61	0.51	0.59	0.65	0.76	0.63
Ca	3.08	0.56	0.33	0.19	0.68	0.66	0.45	0.48
K	0.90	0.17	0.56	0.42	0.29	0.28	0.60	0.47

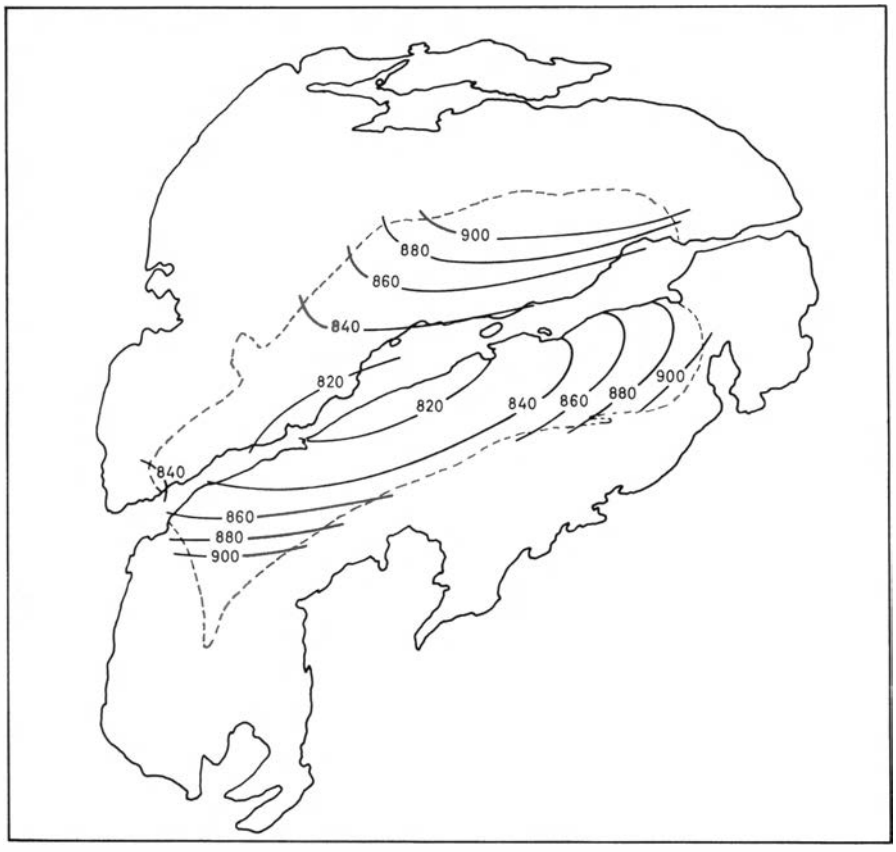

Fig. 3. Si isopleth map of Langø and Gumø. Numbers are counts per second of Si specific radiation. Map based on separate calculations for each of the islands displays excellent agreement of contours. 2nd degree polynomial.

64.65°. The equations which may be interpreted as above for Z-values 2000, 2400, 2600, 2800, 3000, and 3100 are shown in Fig. 4.

The 4th degree equations for Ti will not be given here because of their length. They are shown for Z-values 800 to 1800 in Fig. 5.

It appears from Figs. 3–5 that there is good agreement between the gabbros at Gumø and Langø as far as the isopleth equations for the elements Si, Ti, and Fe are concerned.

Test of apparent Si enrichment at the border of the gabbro

We have also tested the assertion that the gabbro is more acid along the border and that there is a locus of low Si-values at the center of the body. We suppose that the Si surface for both islands can be described by a polynomial of 2nd degree; that is

$$Z = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 xy + \beta_5 y^2 + e,$$

where (x, y) are the geographical coordinates of a sample in the gabbro and Z is the X-ray intensity of Si radiation of that sample; e is the random error.

So that this relation represents an ellipse with axes increasing with Z, two conditions are necessary and sufficient:

(5) $\beta_3\beta_5 > (\frac{1}{2}\beta_4)^2$ and $\beta_3 + \beta_5 > 0$.

The estimate of the relation based on the data is

$$Z = 1048.4 - 67.26x - 1.80y + 5.50x^2 + 1.16xy + 1.04y^2 + e$$

and we see that both conditions for this estimated relation are fulfilled, but this may be due to chance.

Although possible, we have not tested the first condition because the test requires a lot of computations (see Spjøtvoll 1969). However, the difference

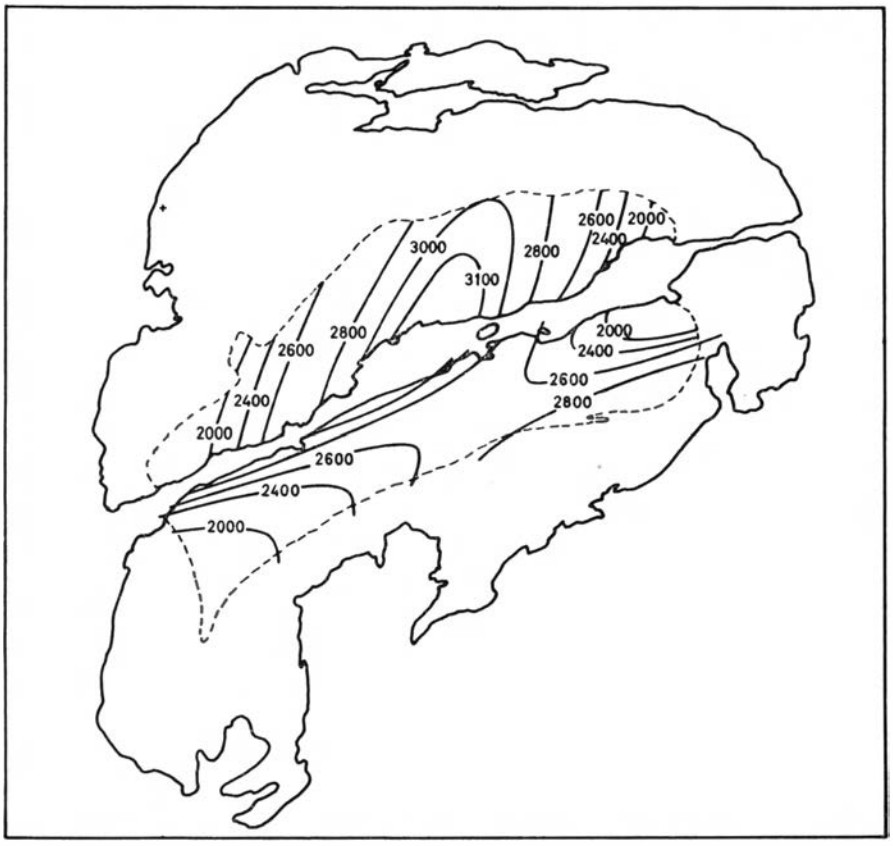

Fig. 4. Fe_{tot} isopleth map of Langø and Gumø. Numbers are counts per second of Fe specific radiation. Map based on separate calculations for each of the islands displays excellent agreement of contours. 2nd degree polynomial.

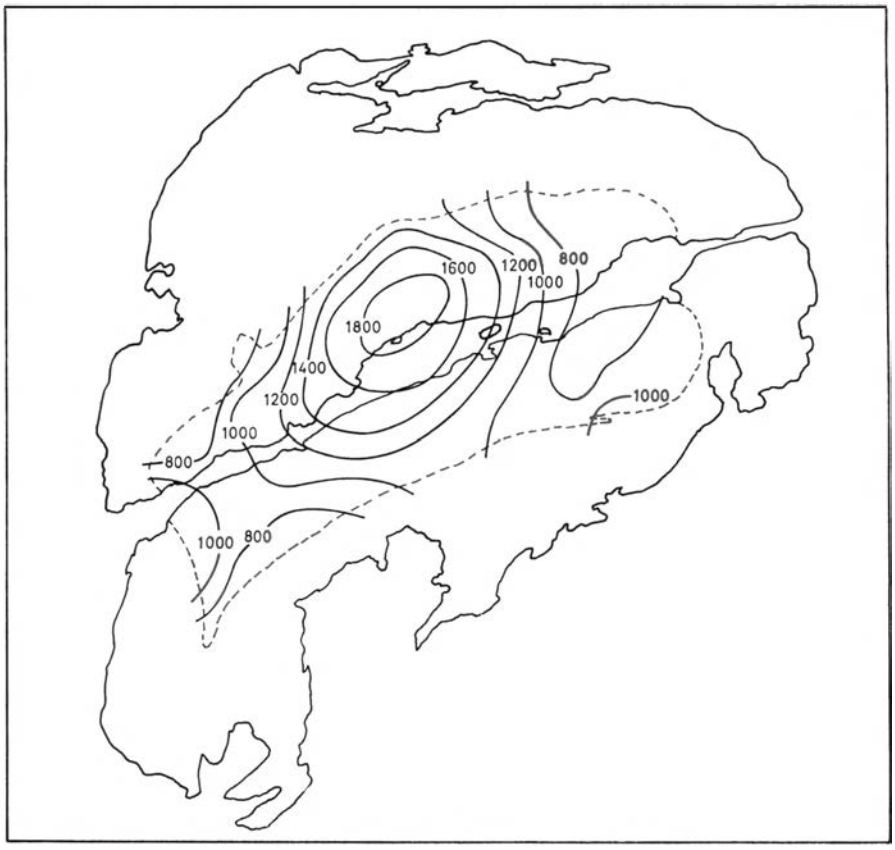

Fig. 5. Ti isopleth map of Langø and Gumø. Numbers are counts per second of Ti specific radiation. Map based on data from both islands. 4th degree polynomial. Contours indicate an elongated area of high values on Langø. This area corresponds to an area of considerable Ti enrichment found by Wiik (1962).

$\hat{\beta}_3\hat{\beta}_5 - (\frac{1}{2}\hat{\beta}_4)^2 = 5.38$ is so large compared with the standard deviations of the regression coefficients involved (1.04, 0.54, 1.30) that it is probably highly significant. The second condition may be tested by an ordinary t-test. The estimate of $\text{var}(\hat{\beta}_3 + \hat{\beta}_5) = \text{var}\hat{\beta}_3 + \text{var}\hat{\beta}_5 + 2\text{cov}(\hat{\beta}_3, \hat{\beta}_5)$ equals 1.39 and this gives the t-statistic (which has 68 degrees of freedom) equal to 4.7 which is highly significant.

We therefore conclude that the data indicate increasing Si concentration from the center to the border of the gabbro.

Dependence between elements on Langø and Gumø

Our last point in the statistical analysis is a simultaneous consideration of the elements. We want to know whether the dependence between the elements is about the same for Gumø and Langø.

Consider the N samples from one of the islands ($N = n_1$ and n_2). We change our earlier notations and define

U_{1k} = X-ray intensity of Si from sample no. k
U_{2k} = X-ray intensity of Ti from sample no. k
U_{3k} = X-ray intensity of Al from sample no. k
U_{4k} = X-ray intensity of Fe from sample no. k
U_{5k} = X-ray intensity of Ca from sample no. k
U_{6k} = X-ray intensity of K from sample no. k

Put

$$\underline{U}'_k = (U_{1k}, U_{2k}, U_{3k}, U_{4k}, U_{5k}, U_{6k}), \quad k = 1, \ldots, N$$

Then, $U_1, \ldots, U_N$ are N stochastic vectors which we assume satisfy the following model

(6) $\underline{U}_1, \ldots, \underline{U}_N$ are independent and multinormally distributed with covariance matrix $\underline{\Sigma}$.

(7) The expectation of U_k may be written $E\underline{U}_k = \underline{\beta}\underline{t}_k$ where $\underline{\beta}$ is a $(6 \times (r + 1))$ matrix with regression coefficients and $\underline{t}_k = (1, x_k, y_k, \ldots, y_k^g)$.

This model is a generalization to the multidimensional case of the model (1) – (3) and is described for instance in Anderson (1957, chap. 8). The dependence between the elements is expressed in the covariance matrix $\underline{\Sigma}$ which is estimated by

$$\underline{\hat{\Sigma}} = \frac{1}{N-(r+1)} \sum_{k=1}^{N} (\underline{U}_k - \underline{\hat{\beta}}\underline{t}_k)(\underline{U}_k - \underline{\hat{\beta}}\underline{t}_k)'$$

where

$$\underline{\hat{\beta}} = \underline{C}\underline{A}^{-1}, \quad \underline{C} = \sum_{k=1}^{N} \underline{U}_k \underline{t}'_k, \quad \underline{A} = \sum_{k=1}^{N} \underline{t}_k \underline{t}'_k$$

Let us denote by $\hat{\sigma}_{ij}$ the estimated covariance between the elements i and j. Then $\underline{\hat{\Sigma}} = \{\hat{\sigma}_{ij}\}$ and the correlation coefficients between these two elements are estimated by

$$\hat{\varrho}_{ij} = \frac{\hat{\sigma}_{ij}}{\sqrt{\hat{\sigma}_{ii}\hat{\sigma}_{jj}}}$$

This estimate is significant if

$$\sqrt{N-(r+1)}\,|\,\hat{\varrho}_{ij}|/\sqrt{1-\hat{\varrho}_{ij}^2} > t_{N-(r+1)-1,\,1-\varepsilon/2}$$

where the number on the right side is the upper $\varepsilon/2$ point in the Student distribution with $N-(r+1)-1$ degrees of freedom.

When applying the above equations with $r=5$ on the data, we obtain the correlation coefficients of Tables 3 and 4.

A correlation from Gumø is significant if its absolute value exceeds 0.34. For Langø the corresponding number equals 0.37 (5% significance level). The agreement between the correlations is remarkably good. Although we have not used any statistical method to test if the correlation tables are different, it is quite clear that our data will not support such a hypothesis.

The results from the study of the dependence between the elements are therefore in accordance with the results from the regression analysis for each element: in none of the cases do our data indicate any geological difference between the gabbros of Langø and Gumø.

Table 3. Correlation coefficients for Gumø. Model of 2nd degree.

Element	Si	Ti	Al	Fe	Ca	K
Si	1.00	0.05	0.00	–0.25	–0.21	0.36
Ti		1.00	–0.68	0.77	–0.42	0.22
Al			1.00	–0.80	0.33	0.02
Fe				1.00	–0.44	0.01
Ca					1.00	–0.42
K						1.00

Table 4. Correlation coefficients for Langø. Model of 2nd degree.

Element	Si	Ti	Al	Fe	Ca	K
Si	1.00	–0.11	–0.27	–0.22	–0.39	0.09
Ti		1.00	–0.64	0.71	–0.47	0.04
Al			1.00	–0.66	0.60	–0.10
Fe				1.00	–0.33	–0.08
Ca					1.00	–0.41
K						1.00

Discussion

In the field the intrusive character of the Langø-Gumø gabbro is obvious and has never been questioned in the literature. The intrusive mechanism of basic melts is under discussion, and the present work does not give sufficiently detailed data to contribute to that discussion. Still our data invite the presentation of a petrogenetic hypothesis:

The systematic geochemical variations presented in the isopleth maps are likely to be the result of some differentiation process active in the magma before or during the emplacement. The magma may have consisted of a considerable amount of unmelted material, and still chemical differentiation by gravity is likely to have taken place because of the chemical difference between solid and melt.

Melting, partial or complete, produces a mass that is less viscous and less dense than the corresponding solid rock, and intrusion may be the result of a density contrast between solid rock and magma, as shown experimentally by Ramberg (1967) and Elder (1970).

These experimental results imply that a dome or neck supplied from a chemically stratified mass will have a concentric structure of concentration isopleths. The gradient of the structures depends on the gradients in the mass before the intrusion. Elements concentrated along the borders of the intrusive body are likely to have been enriched at the top of the magma, and those concentrated at the center of the intrusive body are likely to have been concentrated at the bottom.

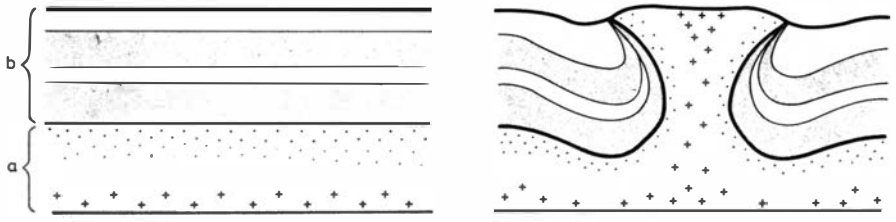

Fig. 6. Very simplified model of intrusion of a differentiated magma (a) intruding into overlying rocks (b). Si enrichment indicated by dots, Ti enrichment by crosses. *Left:* before intrusion; *right:* after intrusion.

If the variation of chemical components is due to such a mechanism of differentiation, the results of the statistical analysis of the X-ray intensity data of the Langø-Gumø gabbro indicate that the magma from which the gabbro was supplied was enriched in SiO_2 at the top and in TiO_2 at the bottom, as displayed in the very simplified sketch (Fig. 6).

For the other elements studied, the picture is not quite so clear, either because the chemical differentiation was less extreme during the emplacement or because they have been disturbed by post-intrusive metamorphism.

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