How important are elastic deflections in the Fennoscandian postglacial uplift?

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When a force is applied to the Earth's surface, there is an immediate elastic deformation, proportional to the stress. This paper reports the calculations of the elastic deflections caused by the unloading of the Late Weichselian ice sheets in Fennoscandia. The deflections caused by the unloading of the maximum Late Weichselian ice sheet are calculated to be 76 m in the central parts of the former glaciated area. This is less than 8% of the isostatic response in the same area. When taking into consideration that this displacement is gradually recovered as the Earth readjusts towards isostatic equilibrium, the elastic mechanism can be assumed to be of little importance for the overall postglacial uplift. Maximum elastic deflection in Fennoscandia is calculated to be a subsidence of 12 m in the central parts of the former glaciated area, and 4–5 m along the Norwegian coast, over the last 10,000 years.

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Introduction

The postglacial uplift has been modelled by various modellers with different, more or less advanced, program algorithms. In most cases the models include the viscous and elastic properties of the Earth. A short overview of different models used to study the Earth's response to glacial loads can be found in Wolf (1994).

In some cases, however, the elastic deflections are specifically calculated. Slichter & Caputo (1960) calculated a central elastic displacement of 0.03 H of a cylindrical square-edged load 450 km in radius. H is the height of the uniform thickness ice load. Cathles (1971) found that a 62-m elastic uplift would be expected upon the removal of the Fennoscandian ice sheet (1700 m thick, with radius 550 km). The elastic displacement is gradually recovered as the Earth adjusts toward isostatic equilibrium. When isostatic equilibrium is achieved, there is no elastic deformation. Cathles (1971) also estimated the elastic response of the center of a cylindrical depression from equilibrium position to be 12% of the isostatic displacement. In other words, as a rough estimation, the total uplift that has occurred since a specific time, and the rate of uplift

Table 1. Shear rigidity μ (10^{11} N/m^2) as a function of depth (From Bullen 1965).

Depth (km)	μ
30	0.63
100	0.67
200	0.74
300	0.81
400	0.90
500	1.10
600	1.32
800	1.69
1000	1.89
1400	2.15
1800	2.39
2200	2.63
2600	2.88
2900	3.03

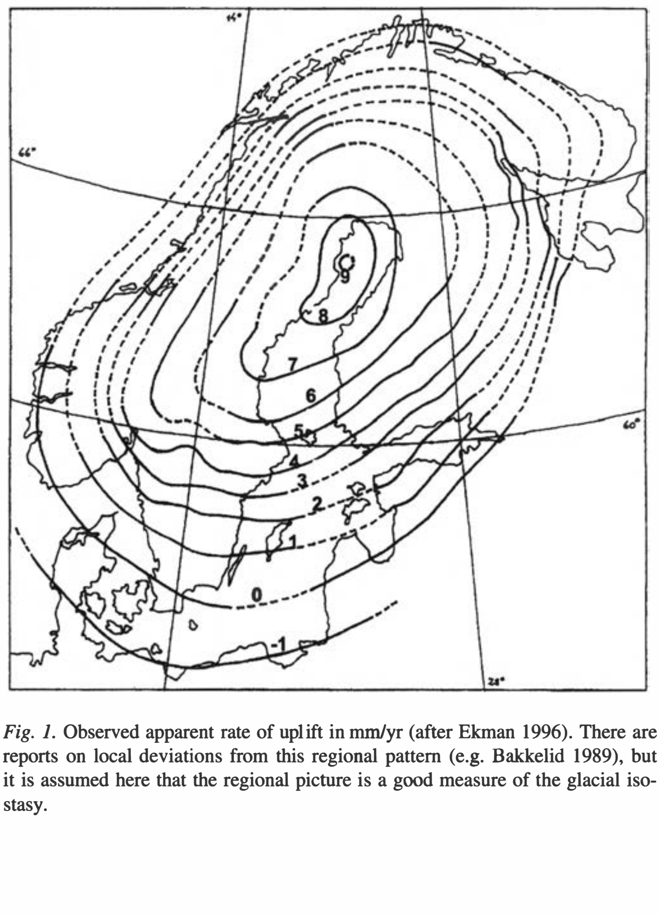

Fig. 1. Observed apparent rate of uplift in mm/yr (after Ekman 1996). There are reports on local deviations from this regional pattern (e.g. Bakkelid 1989), but it is assumed here that the regional picture is a good measure of the glacial isostasy.

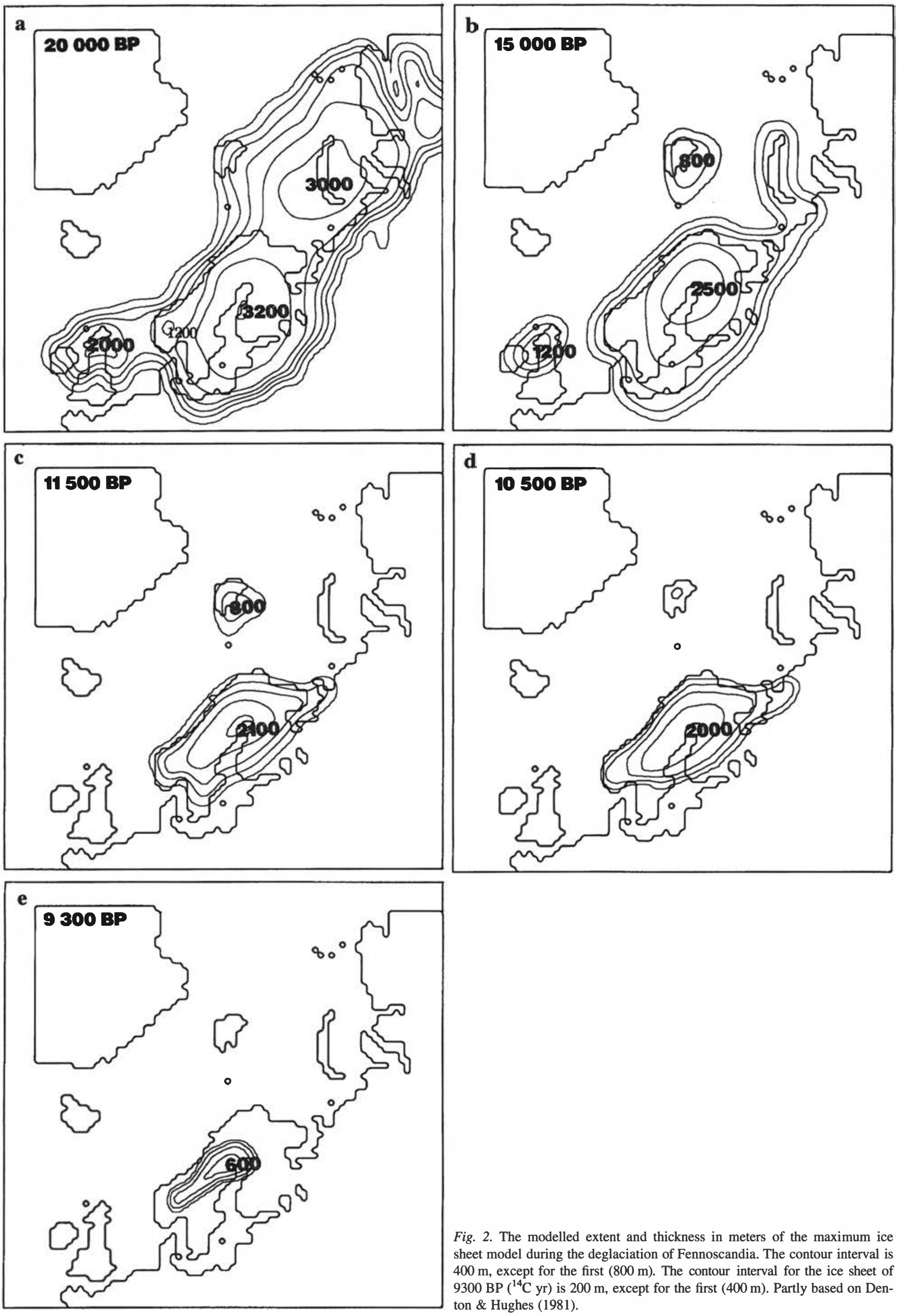

Fig. 2. The modelled extent and thickness in meters of the maximum ice sheet model during the deglaciation of Fennoscandia. The contour interval is 400 m, except for the first (800 m). The contour interval for the ice sheet of 9300 BP (^{14}C yr) is 200 m, except for the first (400 m). Partly based on Denton & Hughes (1981).

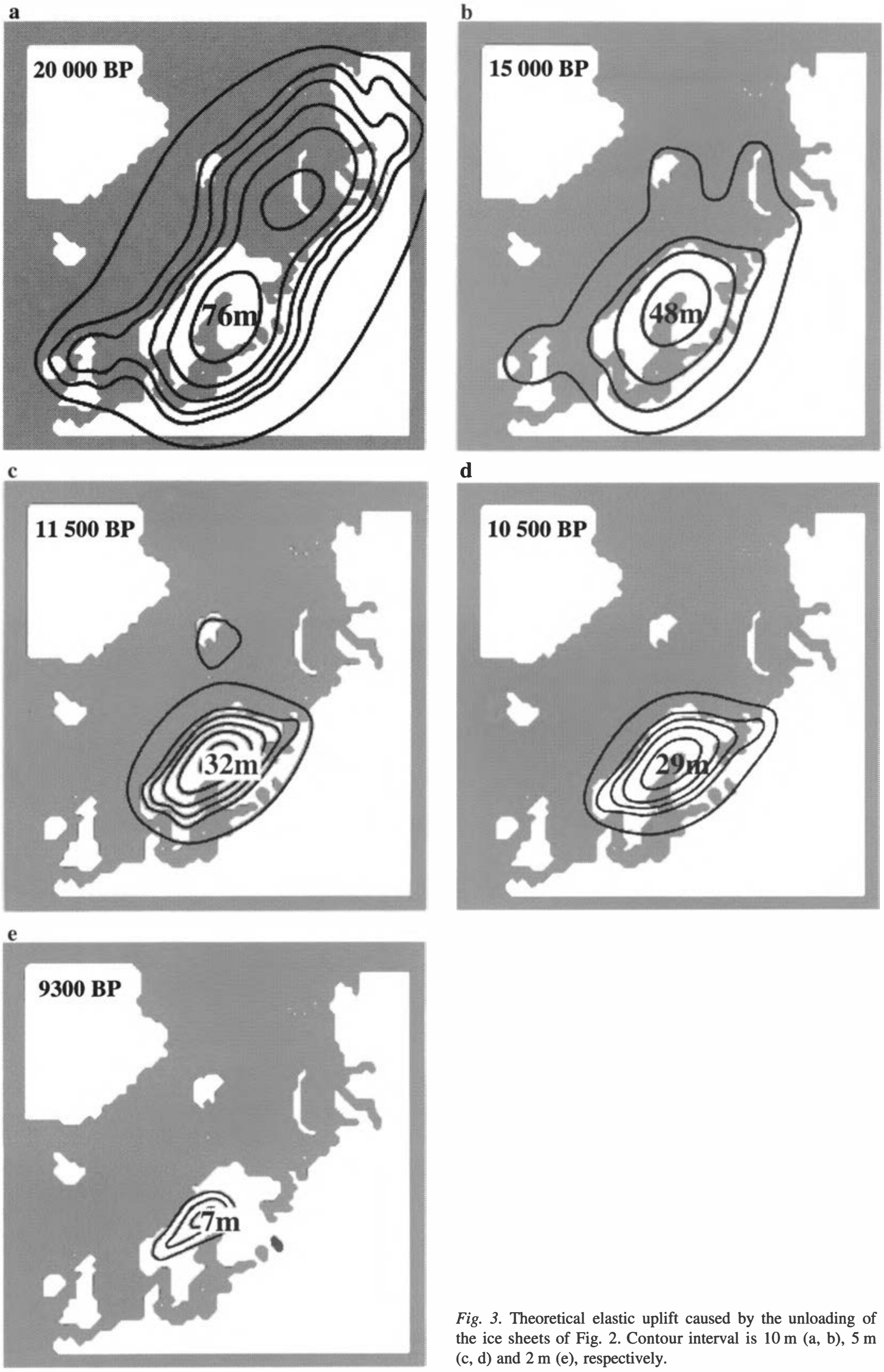

Fig. 3. Theoretical elastic uplift caused by the unloading of the ice sheets of Fig. 2. Contour interval is 10 m (a, b), 5 m (c, d) and 2 m (e), respectively.

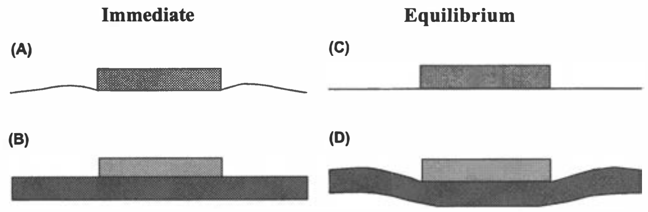

Fig. 4. Illustration of the interrelation between isostatic and elastic deflections. (A) Immediate elastic deflection, (B) immediate isostatic deflection, (C) equilibrium elastic deflection after infinite time, and (D) equilibrium isostatic deflection after infinite time.

occurring at a specific time must be augmented by about 12% to account for the elastic deformation.

In this study the elastic deflection caused by the unloading of the Late Weichselian ice sheets of Fennoscandia is calculated, more realistically, based upon the mapped deglaciation.

Elasticity

When a force is applied to the Earth's surface, there is an immediate elastic deformation, proportional to the stress. Almost all solid rocks behave elastically when the applied forces are not too large, and return to their original shape when the force is removed. The elasticity of a crystalline solid arises from the action of interatomic forces, which tend to maintain each atom in its equilibrium lattice position. A crystal is a stable configuration of the equilibrium positions of atoms. The position of atoms is the result of a balance of attractive and repulsive forces, and the equilibrium spacing is that for which the potential energy of the lattice is minimum.

Rocks behave quite differently in response to applied forces, depending on the elastic properties of the rocks. Changes in rock type within the crust, vertically and horizontally, contribute to variations in the response. The elastic behavior of a material can be characterized by specifying the Lamé's parameters (λ and μ). Data on the

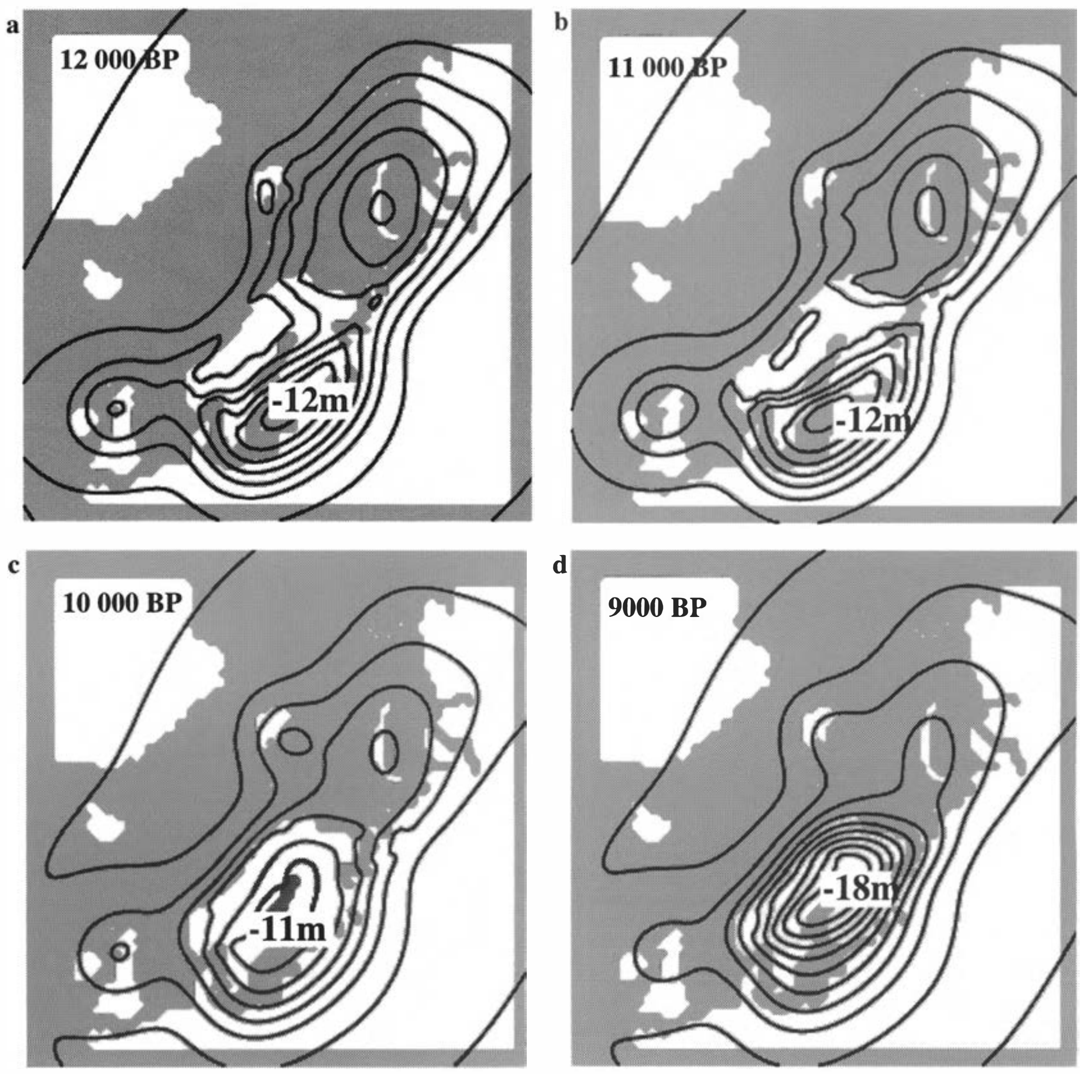

Fig. 5. Theoretical elastic subsidence in meters for late-/ postglacial time. (a) 12,000 BP, (b) 11,000 BP, (c) 10,000 BP, (d) 9000 BP. Contour interval is 2 m.

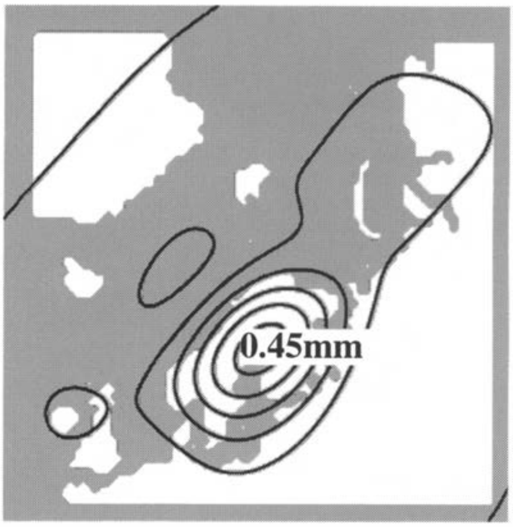

Fig. 6. Theoretical present rate of elastic subsidence, caused by the isostatic uplift of the area. Contour interval is 0.1 mm/yr.

elastic properties of the crust are obtained from several sources, e.g. from seismic velocities and laboratory experiments. The value of the shear rigidity μ as a function of depth into the Earth is shown in Table 1.

Glacial isostasy

After the immediate elastic deflection, there will, by loading/unloading, be a time-dependent isostatic movement to establish a new state of equilibrium.

Based upon modelling of the glacial isostasy, it has been shown in two recent publications by this author (Fjeldskaar 1994, 1997) that the viscosity of the mantle is 10^{21} Pa s overlain by a 75 km asthenosphere with a viscosity of 1.3×10^{19} Pa s. It is generally accepted that the lithosphere behaves like an elastic plate, overlying a viscous fluid. It is further shown (Fjeldskaar 1997) that the flexural rigidity of the lithosphere, which is a measure of the elastic strength, is close to 10^{23} Nm (corresponding to an effective elastic thickness of the lithosphere of $t_e = 20$ km). These conclusions are based upon calibration with the observed tilts of paleoshorelines and the present rate of land uplift (Fig. 1). The modelling is further based upon the assumption that the observed postglacial uplift is a phenomenon caused by glacial isostasy.

The question to be answered is this: are the elastic deflections of such a magnitude that they will significantly affect the postglacial uplift, and thereby alter the above established Earth rheology for this area?

Elastic modelling

Elastic deflection is a function of the wavelength of the load, and also of the elastic properties of the crust. The expected elastic response *u(k)* of an incompressible elastic medium to surface load of wave number *k* is (Cathles 1975):

$$u(k) = \frac{\rho_o \cdot g \cdot h}{\mu \cdot k}$$

where μ is the shear rigidity, g is the gravity, h is the thickness of the load, and ρ_o is the density. In this study the rigidity-depth variation is taken into account, which makes the equation somewhat more complicated (cf. Appendix).

The melting of the Fennoscandian ice sheet is assumed to take place according to the scheme in Fig. 2. The elastic deflection is calculated for the five scenarios (Fig. 3). Unloading of the maximum glaciation ice sheet (20,000 ^{14}C yr BP) can produce an elastic uplift of maximum 76 m in the central parts of the former glaciated area. Unloading of the Younger Dryas ice sheet (10,500 ^{14}C yr BP) could give an elastic uplift of magnitude 29 m (Fig. 3d). In the peripheral parts of the former glaciated area, the elastic deflection will be much less.

The calculations above are only valid for a rigid Earth, but give a rough estimate of the magnitude of elastic deflection. With the introduction of isostatic displacements, the resulting elastic deflections will be different. There are basically two causes of elastic effects: (1) loading/unloading of ice caps, (2) isostatic movements caused by loading/unloading. This is illustrated in Fig. 4.

At the loading of the ice sheet, there will be an immediate elastic response (A). The immediate isostatic response (B) will be close to zero, because of a finite viscosity of the mantle. After infinite time, the lithosphere reaches an equilibrium isostatic position (D), which means that the surface load is balanced by buoyant forces. Lower parts of the Earth will, thus, not 'see' any loads at the surface. The elastic deflection will, at isostatic equilibrium, be zero (C).

The glacier ice started to melt at 20,000 BP, and the area was assumed ice free by 8500 BP. When the ice started to melt, the area would no longer be in isostatic equilibrium. The isostatic response will be a consequence of mantle flow to achieve a new isostatic equilibrium. This response will be delayed in time, compared to the ice melting, and will reduce the elastic deflections. The magnitudes of the elastic deflections in late-/postglacial time are thus expected to be strongly augmented by the isostatic response.

When the isostatic effect is taken into account, the elastic deflections in late-/postglacial time will be significantly different compared to the cases calculated above (Fig. 3). This is because the crust is far from isostatic equilibrium in late-/postglacial time. The maximum elastic deflections from late-glacial time will take place in the southern part of the Baltic Sea (Fig. 5). The area will not be elastically uplifted, but is subsiding, with a magnitude of less than 20 m. Towards the end of the glaciation, the maximum elastic deflections will be moving to the north. Along the coast of Norway, the elastic effect will be a subsidence of only 4–5 m in late-glacial time. The pattern of the present rate of elastic deflection will be similar to the present rate of total uplift of the area, but in an opposite direction; an elastic subsidence with a magnitude of 0.45 mm/yr (Fig. 6).

The parameter input with the most significant uncertainty is the ice thickness, because direct geological evidence is scarce. The ice thickness used here is, probably, a maximum model. However, the same ice thickness model is used for previous isostatic modelling. In comparison with the isostatic effect, the elastic effect is insignificant.

Conclusions

Loading and unloading of the Fennoscandian ice sheets during the last glaciation has resulted in elastic deflections. The deflections caused by the unloading of the maximum Late Weichselian ice sheet are calculated to be 76 m in the central parts of the former glaciated area. This is less than 8% of the isostatic response in the same area. Unloading of the Younger Dryas (10,500 BP) ice sheet gives elastic deflections of maximum 29 m. The isostatic uplift will alter the elastic deflections. When this is taken into consideration, it is reasonable to conclude that the elastic deflection will be insignificant, compared with other uncertainties in the observational data and calculation. Certainly, it is insignificant if dealing with tilts of paleoshorelines along the Norwegian coast. It is thus not important to take the elastic deflections into account in models to simulate the postglacial uplift.

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Appendix

Response of a layered elastic half-space.

The equation of motion for an elastic medium in the gravity field is (Cathles 1975):

$$(1) \qquad \nabla \cdot \tau - \rho_o g_o \nabla u_z + g_o \rho_o \nabla \cdot \mathbf{uz} = 0$$

where τ = stress, g_o = gravity, ρ_o = density, $\mathbf{u}$ = displacement, $\mathbf{z}$ = unit vector vertical.

If the material is assumed to be incompressible, then $\nabla \cdot \mathbf{u} = 0$.

The constitutive equation for an elastic solid is:

$$(2) \qquad \tau_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

where τ_{ij} = stress, ε = strain, μ, λ = Lamé's parameters, δ_{ij} = Kronecker delta,

$$(3) \qquad \varepsilon_{kk} = \frac{\partial u_k}{x u_k} \text{ and } \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{x u_i} + \frac{\partial u_i}{x u_j}\right)$$

The constitutive relation and equation of motion is Fourier Transformed to give the following:

$$(4) \qquad \partial_z \begin{vmatrix} \bar{u}_x \\ \bar{u}_z \\ \bar{\tau}_{xz} \\ \bar{\tau}_{zz} \end{vmatrix} = \begin{vmatrix} 0 & -ik_x & \mu^{-1} & 0 \\ -ik_x & 0 & 0 & 0 \\ 4\mu k_x{}^2 & \rho_o g_o ik_x & 0 & -ik_x \\ \rho_o g_o ik_x & 0 & -ik_x & 0 \end{vmatrix} \begin{vmatrix} \bar{u}_x \\ \bar{u}_z \\ \bar{\tau}_{xz} \\ \bar{\tau}_{zz} \end{vmatrix}$$